Errata for ACTEX Study Manual
Exam LC, 2014 Edition
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Chapter 1

(Page C1-2)

Example 1.7 – Given ${}_2p_1 = 0.6$ and $q_1 = 0.1$, find q_2

Answer – We can solve this two ways:

First way:

I. ${}_2p_1 = p_1 \cdot p_2$

II. $p_1 = 1 - q_1 = 1 - 0.1 = 0.9$

III. ${}_2p_1 = p_1 \cdot p_2$ => $0.6 = 0.9 \cdot p_2$ => $p_2 = 0.6667$

IV. $q_2 = 1 - 0.6667 = 0.3333$

Second way:

I. ${}_2q_1 = q_1 + p_1 \cdot q_2$

II. ${}_2q_1 = 1 - {}_2p_1 = 1 - 0.6 = 0.4$

III. $p_1 = 1 - q_1 = 1 - 0.1 = 0.9$

IV. $0.4 = 0.1 + 0.9 \cdot q_2$ => $q_2 = 0.3333$

(Page C1-4 and C1-5)

4) The probability of a person age 2 dying within half a year.

Answer: ${}_{0.5}q_2$

5) The probability of a person age 5 living 3 years and then dying before age 10.

Answer: ${}_{3|2}q_5$

(Page C1-4 and C1-6)

14) Given ${}_3p_2 = 0.8$ and ${}_2q_5 = 0.2$, find ${}_{3|2}q_2$.

Answer: ${}_{3|2}q_2 = {}_3p_2 \cdot {}_2q_5 = 0.8 \cdot 0.2 = 0.16$

Chapter 2

(Page C2-2)

Example 2.3 – The probability of a person age 0 dying between 20 and 30.

Answer: This is $P(20 < t < 30) = S(20) - S(30) = e^{-.02(20)} - e^{-.02(30)} = 0.1215$

(Page C2-6)

Example 2.11 – Calculate the expected number of years survived for a person age 20 if $S(x) = 1 - \frac{x^2}{6400}$ and $x \leq 80$

Answer – This another way of asking for $\overset{0}{e}_{20}$, given survival to age 20. In other words, we are looking for:

$$\frac{\overset{0}{e}_{20}}{S(20)} = \frac{\int_{20}^{80} 1 - \frac{x^2}{6400}\,dx}{0.9375} = \frac{x - \frac{x^3}{19200}\Big|_{20}^{80}}{0.9375} = 36 \Rightarrow$$

Total years survived = 36 + 20 = 56

(Page C2-28)

2) (Fall 2005, 10) You are given the survival function $S(x)$ as described below:

- $S(x) = 1 - \frac{x}{40}$ for $0 \leq x \leq 40$
- $S(x)$ is zero elsewhere.

Calculate $\overset{0}{e}_{25}$, the complete expectation of life at age 25.

A. Less than 7.7
B. At least 7.7, but less than 8.2
C. At least 8.2, but less than 8.7
D. At least 8.7, but less than 9.2
E. At least 9.2

Answer: The question is asking for the expected future lifetime for a person aged 25. However, we know from the above definition of $S(x)$ that the person must die by age 40, since $S(x)$ (i.e. probability) is zero after 40. So we must solve the equation $E[T_{25}] = \dfrac{\int_{25}^{40} S(x)dx}{S(25)}$:

I. $$\int_{25}^{40} 1 - \frac{x}{40} dx = \left[x - \frac{x^2}{80} \right]_{25}^{40}$$

$$\left(40 - \frac{40^2}{80}\right) - \left(25 - \frac{25^2}{80}\right) = 20 - 17.1875 = 2.8125$$

II. $E[T_{25}] = \dfrac{\int_{25}^{40} S(x)dx}{S(25)} = \dfrac{2.8125}{\left(1 - \frac{25}{40}\right)} = \dfrac{2.8125}{0.375} = 7.5$ => The answer is **A**.

(Page C2-39)

19) Should be moved to 17 in Chapter 3

Chapter 3

(Page C3-17)

1) (Fall 2007, 32) You are given the following life table:

x	ℓ_x
45	1,000
46	900
47	700

Deaths are uniformly distributed within each year of age. Calculate ${}_{0.5}p_{45.75}$.

A. Less than 0.92
B. At least 0.92, but less than 0.93
C. At least 0.93, but less than 0.94
D. At least 0.94, but less than 0.95
E. At least 0.95

Answer: This is a fairly straightforward question. All you need to do is use the UDD assumption to fill in some of the missing values in the table in order to calculate ${}_{0.5}p_{45.75} = \dfrac{\ell_{46.25}}{\ell_{45.75}}$.

I. A total of 100 people died between ages 45 and 46 ($d_{45} = 100$). By the UDD assumption three-quarters of that amount died by age 45.75 so $\ell_{45.75} = 1000 - 0.75 \cdot 100 = 925$.

A total of 200 people died between ages 46 and 47 ($d_{46} = 200$). By the UDD assumption one-quarter of that amount died by age 46.25 so $\ell_{46.25} = 900 - 0.25 \cdot 200 = 850$.

II. ${}_{0.5}p_{45.75} = \dfrac{\ell_{46.25}}{\ell_{45.75}} = \dfrac{850}{925} = 0.9189$ => The answer is **A**.

Chapter 4

(Page C4-3)

Remove the following example:

Example 4.6: (Spring 2005, 3, Exam M) For independent lives (35) and (45):

i. ${}_5p_{35} = 0.90$
ii. ${}_5p_{45} = 0.80$
iii. $q_{40} = 0.03$

iv. $q_{50} = 0.05$

Calculate the probability that the last death of (35) and (45) occurs in the sixth year.

(Page C4-32)

14) (Fall 2004, 9) For John, currently 30 years old, the force of mortality is $\mu_x = \dfrac{1}{100-x}$.

For Bob, an independent life also 30 years old, it is known that

- ${}_{10}p_{30} = 0.94$
- ${}_{5}p_{35} = 0.96$

Calculate the probability that at least one of John or Bob will die within 5 years.

A. Less than 0.0895
B. At least 0.0895, but less than 0.0905
C. At least 0.0905, but less than 0.0915
D. At least 0.0915, but less than 0.0925
E. At least 0.0925

Answer: The problem is asking for the value of ${}_{5}q_{30:30}$, where both John and Bob are currently aged 30:

I. ${}_{5}q_{30:30} = 1 - {}_{5}p_{30:30} = 1 - {}_{5}p_{30} \cdot {}_{5}p_{30}$, where one of the probabilities is for John and one is for Bob.

II. For John, the force of mortality follows De Moivre's theorem =>

$${}_{5}p_{30} = 1 - {}_{5}q_{30} = 1 - \frac{5}{100-30} = 1 - \frac{5}{70} = \frac{65}{70} = 0.9286$$

III. For Bob =>

$${}_{10}p_{30} = {}_{5}p_{30} \cdot {}_{5}p_{35}$$

$$0.94 = {}_{5}p_{30} \cdot 0.96$$

$${}_{5}p_{30} = 0.9792$$

IV. ${}_{5}q_{30:30} = 1 - {}_{5}p_{30:30} = 1 - {}_{5}p_{30} \cdot {}_{5}p_{30} = 1 - 0.9792 \cdot 0.9286 = 0.0907$ => The answer is **C**.